**Modeling Working Mechanisms of Human Cochlea**

**2. Models**

**Mathematic model of temporal (default) characteristic (natural) frequencies and spiral angles of human cochlea:**

 (1)

where,

 (2)

at = 0o or 0 radian spiral angles  are clockwise from -630o or - 7/2  (the cochlear apex, 16 Hz) to 360o or 2  (the cochlear base, 32768Hz) around the origin of the system in a top view of cochlea, 0 (90o or 1/2 ) is a spiral period constant of octaves. See Fig. 1 and the text, a base 2 logarithmic spiral function to plot a cochleogram [17].

**Normalized information model of temporal (default) characteristic (natural) frequencies and spiral angles of human cochlea 【17】:**

 (3)

**Temporal characteristic (natural) angular frequencies and spiral angles of human cochlea:**

 (4)

**Geometric radii and spiral angels of human cochlea:**

 (5)

where, in a cylindrical (or polar) coordinate system, Radiuses r represent centers of the basilar membrane in radial direction; r0 is correspondent to  = 0o. Spiral angles are counterclockwise from - 360o (cochlear base, 32768Hz) to 630o (the cochlear apex, 16 Hz) in a top view of cochlea. See Fig. 2 (a base 2 logarithmic spiral function to plot a cochleagram [17]), Fig. 3 and the text.

**Distributed mass, friction or resistance coefficient and tangent tension along spiral angels:**  (6)

 (7)

 (8)

In a cylindrical coordinate system, the subscript t denotes tangent direction, that is normal to the radial direction.

**Distributed exciting force at the basilar membrane covered with connective tissues (cells):**

 (9)

The subscript z and b respectively denote z direction and basilar membrane. See Fig. 3 and the text.

**Distributed force reacting at the hairs from the tectorial membrane:**

 (10)

**The net force at the combination (see the text) in z direction, using equations 9 and 10,**

 (11)

 (12)

t >= 0; and angles  are from -630o or - 7/2  to 360 o or 2 . See Fig. 1, 2 and 3.

**1st kind of boundary conditions:**

 (13)

 (14)

**Initial conditions:**

 (15)

 (16)

In this paper, Bernoulli separation is frequently used to approximate the functions, to obtain our analytic models. Therefore, we assume,

 (17)

From equation 12, we derive,  (18)

For a free vibration, equation 18 reduces to a homogeneous one:

 (19)

or  (20)

**2.1 Resonating Models of the Combination in Basilar Membrane in Temporal Domain**

**Local** **resonating model at a particular encoding angle:**

 (21)

r denotes a resonating angle at that a physical resonance occurs; and e denotes an encoding angle a variant coefficient or parameter not a variable) at that a particular temporal frequency is encoded.

**2.1.1 Homogeneous (Natural or Transit) Models Determined by Temporal Conditions**

Equation 20 becomes,

 (22)

The left of equation 22 is only a function of time and the right is a constant. Let the constant be Kt:

 (23)

From equations 22 and 23, we get a characteristic equation,

 (24)

We denote a natural or characteristic frequency (square),

 (25)

Using geometric model equation 5 and physical model equation 25, we can derive the mathematical model equation 1,

 (26)

where,  (27)

We assume it is roughly a constant.

From equations 24 and 25, we obtain a standard characteristic equation,

 (28)

where, viscous damping factor is,

 (29)

The general solution of equation 28 is (omitting the encoding angle),

 (30)

where, Az,n and Bz,n are constants that are determined with the initial conditions (equations 15 and 16).

When,  (31)

 (32)

Where, a frequency of a damped free (natural) vibration is,

 (33)

And the solution is an under damped oscillation and roughly represents a range of normal hearing.

When,  (34)

The solution is a critical damping oscillation and probably correlates a aural response obtuse.

When,  (35)

The solution is an over-damped oscillation and maybe brings to a range of an aural hinder. We consider the hinder as a hearing loss.

When,  (36)

The solution is an undamped oscillation and is probably one of causes that give rise to tinnitus.

**2.1.2 Non-Homogeneous Models Excited by a Force:**

**A distributed exciting force acting at the combination from equation 11 is assumed:**

 (37)

Where, F denotes a forced driving angular frequency; subscripts F, z, and r denote a force, z direction and resonance respectively; Az,r,F and Frespectively denote a constant and forced angular function. See the text and Fig. 3.

Using equations 18 and 20, in a similar way to section 2.1.1, we can get a local non-homogeneous solution by the force. It is well known that the peak (maximum) value of complex frequency response, quality factor Q occurs at a resonating angular frequency and angle,

 (38)

 (39)

Therefore, the results are similar to that in section 2.1.1.

**2.4 Our Models of the Dominant Travelling Waves in Human Cochlea**  (40)

where, subscript i denotes ith kind of particle, such as Na+, K+, Cl-, OH- and H3O+; m, P, n, q, E, X, B and  respectively denote correspondent mass, pressure, particle volume density, electrical charge, internal electrical field, cross product, internal magnetic field and effective collision frequency [33]. Use approximation (small distance u, see Fig. 5):

 (41)

where,  C, Q and d are respectively the permittivity, capacitance, total electrical charge of a unit volume plasma and distance between the displaced charges units.

 (42)

V denotes the velocity [26].

From Equations 40 - 42, we get,

 (43)

**3. Discussion**

**Intensity levels of classic sound theory:**

 (44)

or  (45)

where I denotes a sound intensity, I0 denotes a reference value; the both are functions of frequencies. The unit is Bel or dB; in information theory, the unit is Hartley.